Sonic band gaps in one-dimensional phononic crystals with a symmetric stub

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(Received 19 November 2007; published 21 March 2008)

The propagation of elastic waves through a one-dimensional chain of beads with grafted stubs is experimentally as well as numerically investigated. The results obtained by both approaches are well correlated and show that the stub introduces a dip in the spectral response of the chain, which is related to the excitation of a stub mode. A parametric study on the stub is carried out and shows which parameters have an effect on the position and the shape of the stub mode. The results allow potential applications for the filtering and the multiplexing of elastic waves.

DOI: 10.1103/PhysRevB.77.104304 PACS number(s): 43.20.+g

I. INTRODUCTION

For many years, the wave propagation in periodic systems has received a great deal of attention. By analogy with the studies driven on photonic crystals,[1–3] many works were conducted on phononic crystals, which designate two- or three-dimensional periodic arrangements of inclusions in a matrix. These heterogeneous materials may exhibit absolute band gaps, under certain conditions where the propagation of elastic waves is forbidden whatever the direction of propagation of the waves is. This property confers to phononic crystals potential applications in various domains, particularly for the guiding and filtering of acoustic waves.[4–12] In the past decade, the transmission of electromagnetic[13–18] and acoustic[19] waves through photonic and phononic crystals was theoretically and experimentally studied. Linear and different periodically stubbed waveguides[11,15–18] structures were considered.

In the case of photonic crystals, Danglot *et al.*[18] reported experimental and theoretical investigations of a T-stub structure patterned in a two-dimensional photonic crystal. Transmittivity properties are interpreted in terms of multimode propagation within the T-stub region. Results on the propagating eigenmodes in this structure are given as a function of the stub length. Each minimum in the power spectra corresponds to an eigenmode of the photonic stub; thus, propagation toward the output terminal is forbidden. This phenomenon is analogous to the one observed in T-shaped quantum wire structures.[20] Both theoretical and experimental results were also reported about the transmission in coaxial waveguides with laterally attached stubs,[15–17] showing, in particular, the possibility of superluminal or subluminal propagation of electromagnetic waves.

Several authors[6–9] have studied the propagation of acoustic and elastic waves through a structure made of an infinite linear waveguide on which stubs are grafted periodically. Very large band gaps were obtained and by an appropriate choice of the geometry and of the materials constituting the waveguide and the stubs, tunability of the complete spectral gap has been investigated. Similar studies were also conducted by Depollier *et al.*[21] on periodic two-dimensional lattices of slender tubes.

Kafesaki *et al.*[22] applied the finite difference time domain method to investigate the guiding through rectilinear defects created by removing a row of cylinders inside a two-dimensional (2D) phononic crystal made of solid constituents. Similar studies were done by Khelif *et al.*[4,23] in 2D phononic crystals composed of fluid and mixed solid-fluid constituents. References 4, 11, and 23 also showed that a stub resonator connected perpendicularly to a straight waveguide inside a 2D phononic crystal leads to narrow dips in the transmission spectrum of the phononic crystal based structure. The length as well as the width of the stub is varied by removing elements in the periodic arrangement. These "zeros" of transmission can have potential applications in filtering (selective transmission of frequency) and demultiplexing phenomena. In a recent paper, Pennec *et al.*[10] theoretically investigated the properties of acoustic waves through waveguides constituted of steel hollow cylinders incorporated in a phononic crystal made of filled steel cylinders in water. They showed that the transmitted frequency response can be adjusted by selecting the inner diameter of the hollow cylinders or the nature of the liquid filling the tubes. They also discussed the transmission properties of Y-shaped wave guides constituted of hollow cylinders.

The work proposed in this paper follows the spirit of the previous investigations on T-stub waveguides created inside 2D phononic crystals. While in 2D phononic crystals, the waveguide and the stubs were created by removing a row and some elements in the periodic arrangement, in the present work the waveguide is made of a linear chain of glued metal beads, and the stub is created by placing additional beads grafted on the waveguide. This study follows previous works achieved on monoatomic and diatomic chains of beads;[24,25] in particular, localized modes were observed in the forbidden band when the chain is made by the periodic alternation of two different beads. In the present paper, the transmission of an acoustic pulse through a chain is studied; the chain is made of identical beads, and a double symmetric T-stub is considered (to avoid bending motions of the chain). The experimental data are compared with the numerical results, which are obtained with the help of the finite element method or using a simple atomic model. In the first part, the linear chain without a stub is considered. Then, a symmetric stub is grafted at the middle of the chain. Finally, we investigate the transmission when the stub contains beads of various sizes and numbers.

II. LINEAR CHAIN WITHOUT A STUB

In this section, a finite linear chain of $N_c=5$ identical and glued beads is considered. The diameter of each bead is equal to 10 mm. In the theory, it is assumed that each contact between the beads is identical. The chain is excited by a longitudinal force applied at one extremity. The experimental setup has been previously described[24] and is not recalled in this paper. The finite element method, with the help of the ATILA code,[26] is used, and the harmonic analysis gives the displacement at each node of the finite element mesh for each frequency given in the calculation. Thanks to the axisymmetrical axis of the chain, the mesh is limited to the half section. The study concerns the first few modes that correspond to the acoustical branch of the chain. In the case of a chain made up of five identical beads, Fig. 1(a) presents the experimental power spectrum (in decibel) defined as the square of the Fourier transform of the signal measured by the receiver. For a linear approximation, the pressure is proportional to the frequency multiplied by U_x in the case of harmonic displacement. Therefore, the numerical variations of frequency times displacement are plotted in decibel in Fig. 1(b) for a comparison with experimental results that measure the acoustic pressure. Both of these plots are normalized and clearly show several peaks at nearly the same position in frequency. On the one hand, the numerical peaks are sharper than the experimental ones because in the numerical calculation, the losses are not taken into account. On the other hand, experimental peaks are wider because the frequency resolution of the experimental setup is 2 kHz. Small discrepancies in the position of the peaks are probably due to the coupling between the beads that is assumed to be identical in the numerical model. By considering five beads with both ends free, which corresponds to experimental conditions, a previous study relying on a chain of rigid atoms[24] has shown that five discrete frequencies are obtained, whose wave numbers k are equal to $n\pi/L$, where L is the total length of the chain and n is an integer (L is approximately equal to $10R$, where R is the bead radius). The corresponding angular frequencies are equal to

$$\omega_n = 2\pi f_n = (4C/m)^{1/2}\sin(kR) \quad \text{with } k = n\pi/10R \quad (n=0-4), \tag{1}$$

where f_n is the corresponding frequency, C designates the coupling constant, and m is the mass of each individual bead.[27] One can notice that the resonance frequency equal to zero is always the solution of the system ($n=0$) and is not reproduced in Fig. 1. Using a normalization on the frequency f_1 of the first mode ($n=1$), the four resonance frequencies deduced from Eq. (1) are $f_1, f_2=1.9f_1$, $f_3=2.6f_1$, and $f_4=3.1f_1$. The positions of the numerical peaks, which are obtained with the finite element method [Fig. 1(b)], are $f_1, f_2=1.9f_1$, $f_3=2.7f_1$, and $f_4=3.4f_1$. Numerical frequencies are very similar to the frequencies calculated by the simple atomic model [Eq. (1)], particularly for the two first modes. For higher frequencies, the chain of beads cannot be represented by a chain of rigid atoms, and more complex phenomena occur.[27]

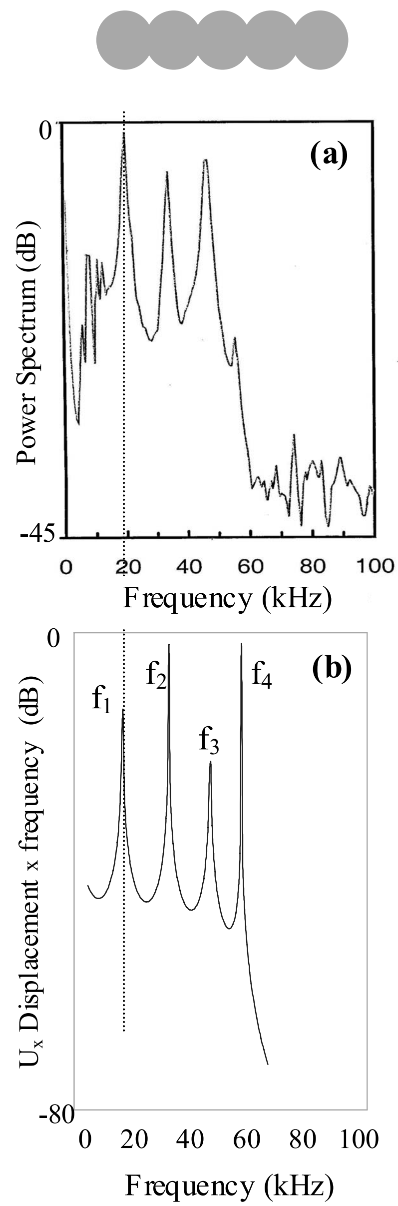

FIG. 1. Chain made up of five identical steel beads 10 mm in diameter. (a) Experimental power spectrum. (b) Numerical curve showing the displacement U_x at the end of the chain multiplied by the frequency as a function of the frequency. Both curves are normalized. The vertical scales are in decibel. The vertical line represents the correspondence between the experimental and numerical peaks.

In the simple atomic model,[24] assuming that the beads oscillate in harmonic motion with the same frequency, the displacement field of the modes can be written as $A\cos(kx)$, where x is the position along the chain and the temporal factor has been omitted for the sake of simplicity. From the numerical calculations, one can see that the normalized amplitudes of the displacements U_x at a given time are quite well reproduced by Eq. (1) (Fig. 2) for the first modes

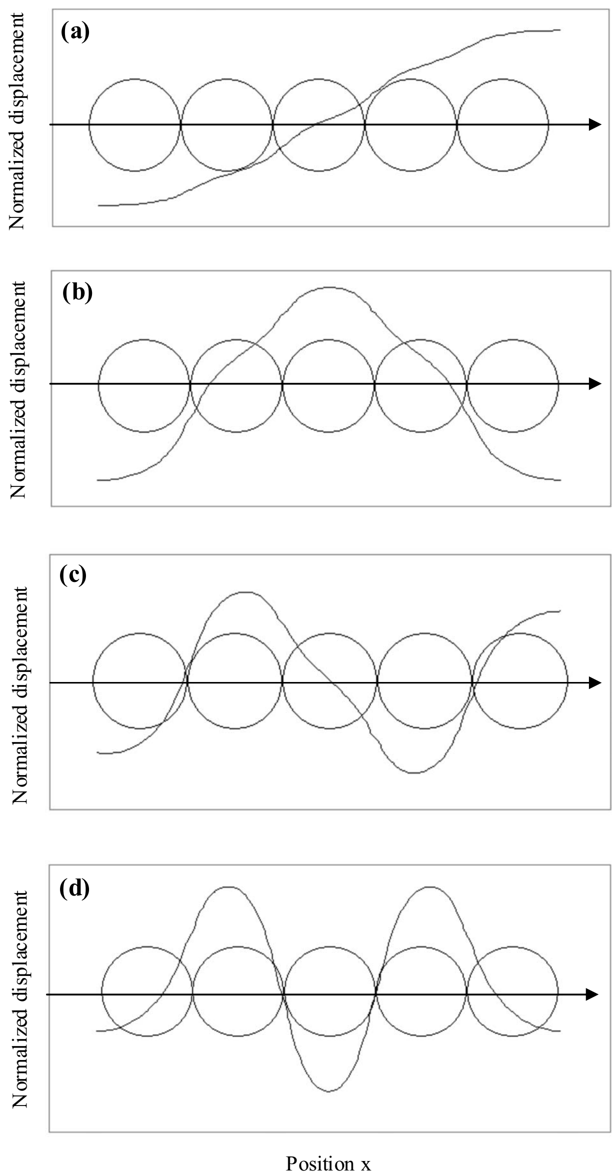

FIG. 2. Chain made up of five identical steel beads 10 mm in diameter. Numerical displacement U_x as a function of the position in the chain for the first modes. (a) Mode $n=1(f_1)$, (b) mode $n=2(f_2)$, (c) mode $n=3(f_3)$, and (d) mode $n=4(f_4)$. The amplitude is normalized. The five circles show the rest position of the beads.

$(n=1-4)$. In Fig. 2, the five circles represent the rest position of the beads. The displacement for the mode at frequency equal to zero $(n=0)$ is not drawn because the related displacement is a simple translation.

(1) The displacement of the chain for mode $n=1$ is presented in Fig. 2(a). It can approximately be written as $A\cos(\pi x/10R)$ and, as expected, the displacement is equal to zero at the middle of the chain $(x=5R)$ and is symmetric with respect to the middle of the chain.

(2) For mode $n=2$, the displacement is presented in Fig. 2(b) and can approximately be written as $A\cos(\pi x/5R)$: It is maximum at the middle of the chain and at the two extremities.

(3) For mode $n=3$, the corresponding displacement is roughly $A\cos(3\pi x/10R)$. It is also equal to zero at the middle of the chain [Fig. 2(c)] and is antisymmetric with respect to the middle of the chain. The first two beads have a displacement in the opposite direction.

(4) For mode $n=4$, the displacement is approximately written as $A\cos(4\pi x/10R)$, as expected, and is maximum at the middle of the chain [Fig. 2(b)].

Finally, Fig. 2 shows that the simple atomic model can be used as a first approximation to describe the motion of the chain of beads although the quantitative behavior is more complex and involves the deformation of the beads.

III. LINEAR CHAIN WITH A SYMMETRIC STUB GRAFTED AT THE MIDDLE OF THE CHAIN

In this section, we symmetrically attach two beads (10 mm in diameter) at the sides of the previous chain made of five beads $(N_c=5)$. The new beads are attached at the middle of the chain. This ramification is named stub, in reference to work on photonic[18] and phononic[4,7] crystals. The total number of beads in the stub (N_s) is equal to 3, including the bead contained in the linear chain $(N_s=3)$. In the present study, the case of a symmetric stub has been privileged to avoid bending motions of the whole chain that are generated with a nonsymmetric stub. The analysis is performed using the previous experimental procedure, as well as simple atomic model and numerical calculations.

First, it is worth noting that in the frame of a simple atomic model considering only the interaction between adjacent beads, a dip occurs in the transmission spectrum at the angular frequency $\omega_{\mathrm{dip}}=\sqrt{C'/M'}$, where C' designates the coupling constant between the atom at the center of the chain and the atom in the stub and M' is the mass of the atom in the stub. This frequency is the resonance frequency of the latter atom when the atom at the center of the chain is rigidly fixed.

The experimental and numerical analyses of the stubbed waveguide constituted by the beads follow the same idea, although the origin of the resonances in the stub is more complex due to the deformation of the beads during their motion.

To limit the time of the numerical simulations, infinite cylinders instead of beads are considered, and the plane strain condition is used. In this way, the discretized mesh is only two dimensional instead of being three dimensional, as in the case of beads. Several numerical tests have been performed to verify this hypothesis and have shown that it is valid using a multiplicative factor on the frequency scale between the bead and cylinder results.

Figure 3 presents the experimental power spectrum of the transmitted signal and the numerical displacement at the end of the chain as a function of frequency. A good agreement is obtained between experimental and numerical results, as concerns the position of the peaks. These curves clearly show a dip in the response, as previously noticed in the simple atomic model.

In the aim of determining the origin of the minimum observed in Fig. 3, the displacement U_x of the chain is presented in Fig. 4 for the main peaks and dip of the curves. The circles represent the rest position of the beads.

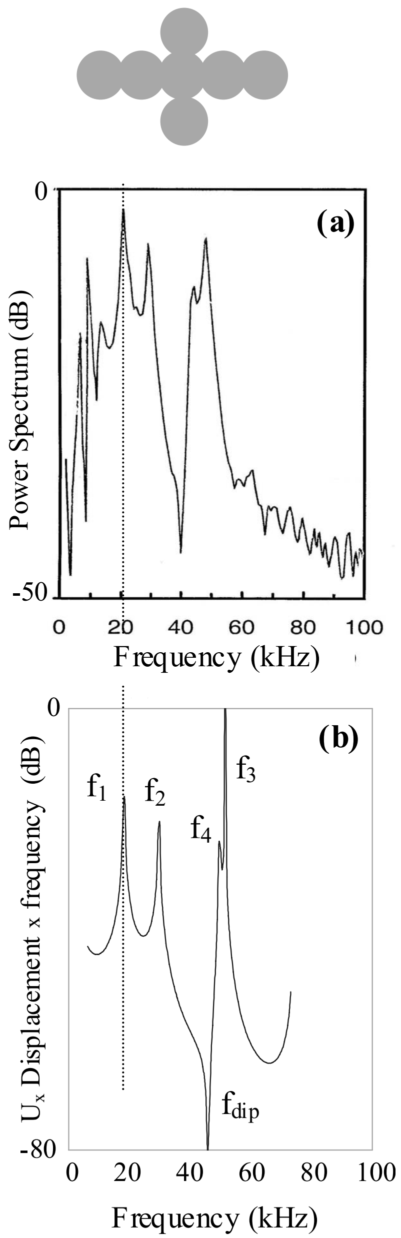

FIG. 3. Chain made up of five identical steel beads, with a symmetric stub grafted at the middle of the chain, all 10 mm in diameter. (a) Experimental power spectrum. (b) Numerical curve showing the displacement U_x at the end of the chain multiplied by the frequency as a function of the frequency. Both curves are normalized. The vertical scales are in decibel. The vertical line represents the correspondence between the experimental and numerical peaks.

(1) A mode at f_1 ($n=1$) is obtained, and the corresponding displacement field [Fig. 4(a)] is very similar to the one obtained without the stub [Fig. 2(a)]. One can notice that introducing a stub does not change the frequency of this mode because the additional beads are attached on a fixed bead: The displacement along the chain is equal to zero for the bead at the middle of the chain.

(2) The frequency of mode $n=2$ is significantly changed by introducing the stub ($f_2=1.6f_1$ instead of $1.9f_1$). The corresponding displacement at f_2 [Fig. 4(b)] is very similar to

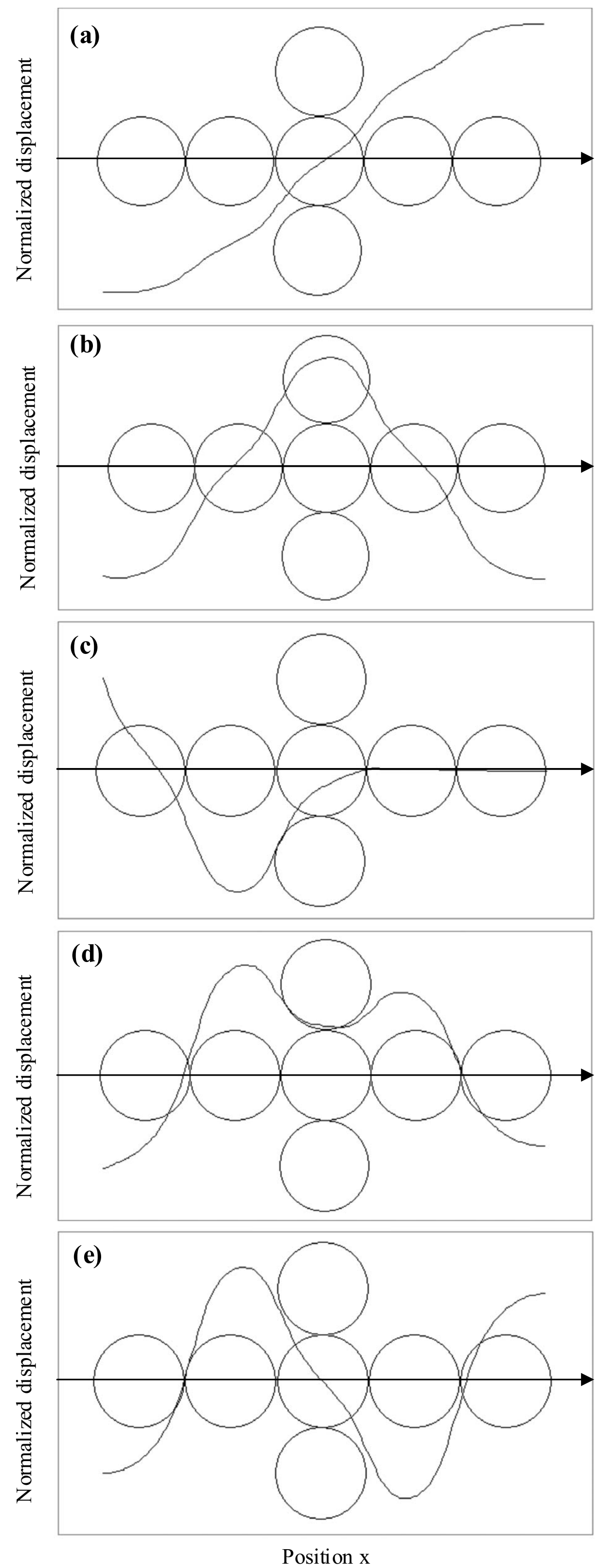

FIG. 4. Chain made up of five identical steel beads, with a symmetric stub grafted at the middle of the chain, all 10 mm in diameter. Numerical displacement U_x as a function of the position in the chain at the following frequencies: (a) f_1, (b) f_2, (c) f_{dip}, (d) f_4, and (e) f_3. The amplitude is normalized. The circles show the rest position of the beads.

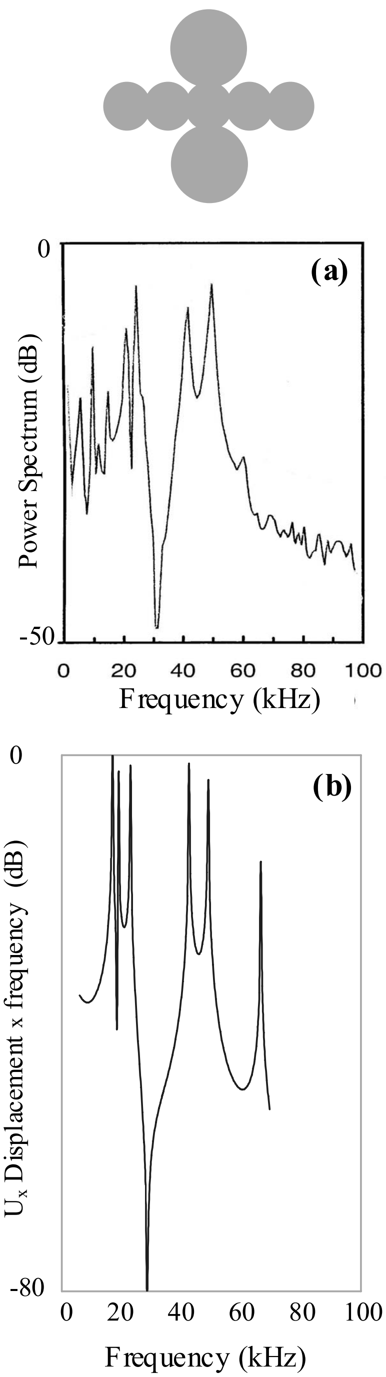

FIG. 5. Chain made up of five identical steel beads 10 mm in diameter, with a symmetric stub grafted at the middle of the chain 15 mm in diameter. (a) Experimental power spectrum. (b) Numerical curve showing the displacement U_x at the end of the chain multiplied by the frequency as a function of the frequency. Both curves are normalized. The vertical scales are in decibel.

the case without a stub [Fig. 2(b)], but the frequency is quite different because the additional beads are attached on a moving bead.

(3) At the frequency of the dip ($f_{dip}=2.5f_1$), the numerical displacement [Fig. 4(c)] shows that the vibration is located in the left part of the chain and in the stub, whereas it is negligible in the right part. Thus, the dip is related to an eigenmode of the stub, which forbids propagation of waves toward the output terminal. The stub has an important influence on the propagation of elastic waves from one extremity to the other: the wave enters the stub, is reflected at the end of the stub, and returns back to the beginning of the chain. Similar trends were obtained in 2D photonic crystals[18] and in 2D phononic crystals containing a stub grafted on a linear waveguide,[6] in analogy to the prediction of a simple atomic model. However, in the latter model, the atom at the center of the chain and the following atoms to the right remain at rest at the frequency of the dip. In the case of the chain of beads, the atom at the center of the chain still displays a small motion that expresses the departure of the quantitative result with respect to that of a simple model.

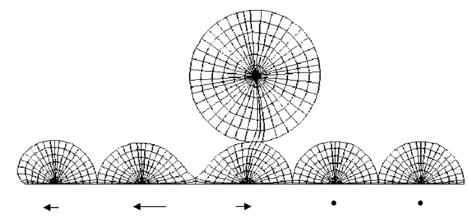

FIG. 6. Chain made up of five identical steel beads 10 mm in diameter, with a symmetric stub grafted at the middle of the chain 15 mm in diameter. Numerical displacement field at the frequency of the dip in the spectrum. The amplitude is normalized. The arrows show the displacement directions. The dots show that the bead does not move.

(4) At $2.7f_1$, a small peak appears in the curves of Fig. 3. The related displacement field [Fig. 4(d)] shows a motion similar to the $n=4$ mode [Fig. 2(d)] but disturbed due to additional beads. The frequency of the mode has drastically decreased ($f_4=2.7f_1$ instead of $3.4f_1$) because the additional beads are attached on the moving third bead.

(5) Figure 4(e) presents the displacement field at $2.8f_1$, which corresponds to the $n=3$ mode. It is very similar to the displacement presented in Fig. 2(c). The frequencies with and without a stub are close because the additional beads are attached on a fixed bead ($f_3=2.8f_1$ instead of $2.7f_1$).

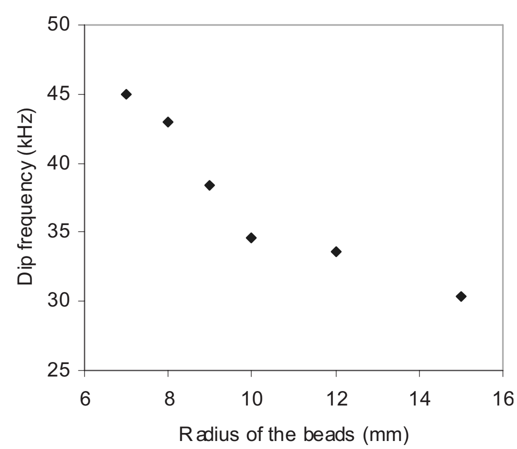

FIG. 7. Variations of the frequency of the dip observed on the experimental power spectra, as a function of the radius of the beads in the stub ($N_c=5$, $N_s=3$).

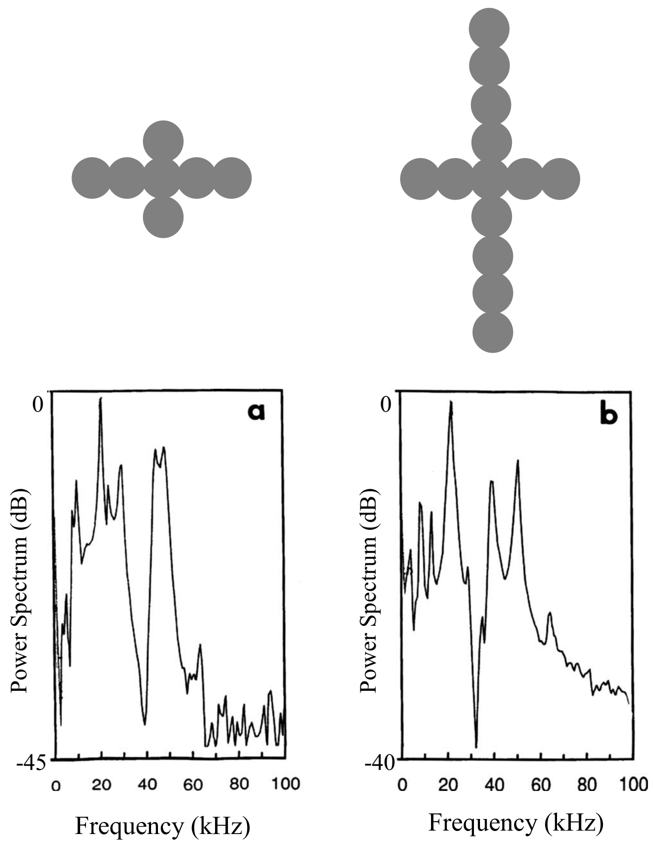

FIG. 8. Experimental power spectrum of a chain (N_c=5) with a stub grafted at the middle of the chain. (a) Three beads in the stub (N_s=3); (b) nine beads in the stub (N_s=9). The vertical scales are in decibel.

More generally, in some frequency range, the stub has a strong influence because it cancels the transmission from one extremity of the chain to the other, whereas its influence is weak in other frequency ranges where the waves travel along the chain without being perturbed by the stub.

IV. PARAMETRIC STUDIES

A. Influence of the diameter of the grafted beads: N_s=3, N_c=5

Studies are performed for several beads in the stub, with the diameter of the additional beads varying from 6 to 15 mm and the stub being still symmetric. The number of beads in the stub N_s and the number of beads in the chain N_c are fixed to 3 and 5, respectively. Figure 5 presents the experimental data and the numerical results as a function of the frequency for grafted beads 15 mm in diameter. The different extrema are identified in the figure. There is a reasonably good agreement between the results in terms of the relative positions of the peaks. The experimental data and the numerical calculations clearly show that with heavier beads in the stub, the dip moves toward lower frequencies, as expected by the simple relation giving the position of the dip with the help of the simple atomic model ($\omega_{\text{dip}}=\sqrt{C'/M'}$). In this case, M', the mass of the bead in the derivation, is higher. Thus, the frequency of the dip is lower than the previous one. At the frequency of the dip ($f=1.5f_1$), the numerical displacement field is drawn in Fig. 6 and shows again that the motion is limited to the left part of the chain. Similar results have been observed on photonic crystals[18] and on arrays of cylinders in water,[4] where a longer stub induces a decrease in the dip frequency.

Figure 7 presents the experimental variations of the frequency position of the dip as a function of the diameter of the bead in the stub. This frequency decreases as the diameter of the beads in the stub increases. However, it was not possible to derive a simple relation describing this behavior due to the complex nature of the resonance frequency of the stub. Nevertheless, this curve can be used for adjusting the frequency of the dip.

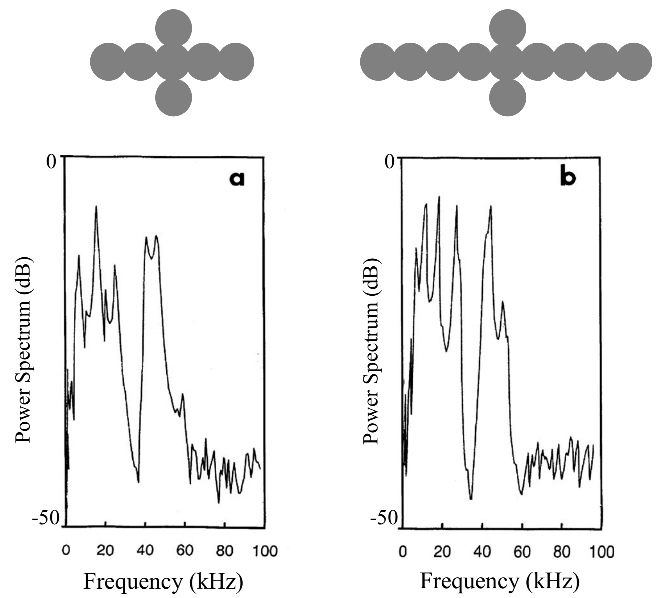

FIG. 9. Experimental power spectrum of a chain with a stub (N_s=3) grafted at the middle of the chain. (a) Five beads in the chain (N_c=5); (b) nine beads in the chain (N_c=9). The vertical scales are in decibel.

B. Influence of N_s, number of beads in the stub, with N_c=5

Experiments have been performed for investigating the influence of the number of beads in the stub on the transmission. The reference case (N_s=3 and N_c=5) is presented in Fig. 8(a). Figure 8(b) presents the experimental power spectrum when the total number of beads in the stub is equal to 9 (N_s=9), grafted at the middle of the chain. A comparison of these curves shows a very small decrease of the dip frequency in the second case, which supports the fact that this frequency is essentially determined by the nature of the first bead in the stub.

C. Influence of N_c, number of beads in the chain, with N_s=3

In this section, experiments have been performed on a longer chain with a symmetric stub. The total number of beads in the stub is 3 (N_s=3), grafted at the middle of the chain. The reference case (N_s=3 and N_c=5) is presented in

Fig. 9(a). Figure 9(b) presents the experimental power spectrum when the length of the chain is equal to 9 ($N_c=9$). The increase in the number of beads in the chain gives rise to new modes, as previously observed,[24] but the position and the shape of the dip are not affected. This again shows the close relationship between the dip and the presence of the stub.

V. CONCLUSION

The propagation of waves along a chain of beads with symmetric stubs, grafted at the middle of the chain, has been performed both experimentally and theoretically. The results have shown that the presence of a stub in the chain introduces a dip in the transmission response, as previously observed in 2D photonic and phononic crystals where the stub was grafted on a linear waveguide.[4,18] The numerical analysis has shown that this dip is due to the excitation of a stub mode that cancels the transmission from one extremity of the chain to the other. The position and the shape of the dip in the response are related to the geometry and nature of the stub: A heavier stub shifts the position of the dip to lower frequencies, whereas a longer stub with identical beads does not significantly change the frequency of the dip because this is mainly determined by the interaction of the waveguide with the first bead in the stub. The results show that it is possible to adjust the position of the dip and it opens potential applications of these structures for filtering or demultiplexing. In this paper, the stub was always grafted at the middle of the chain of beads, but other potentialities can be forecast by changing the position of the stub in the chain as well as by introducing periodically grafted stubs in the chain. Although both the experimental and theoretical investigations reported in this paper belong to the linear acoustic regime, our structure should also be suitable for further studies in the nonlinear regime, implying more intense excitations.[28]

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